

POLITECNICO
MILANO 1863

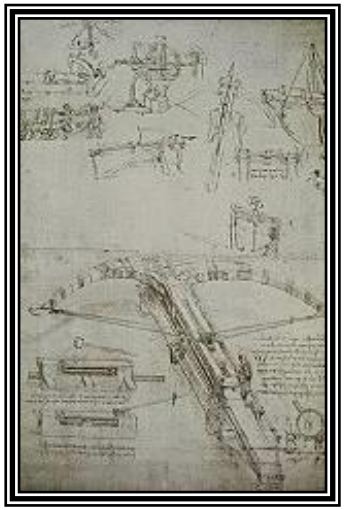
Effetti di scala nelle strutture in calcestruzzo

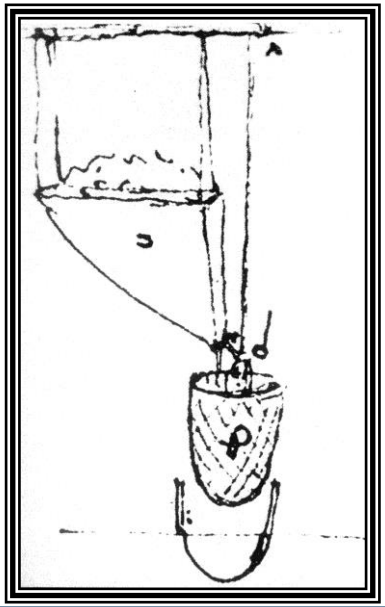
G. ROSATI and G. Muciaccia

LEONARDO DA VINCI *(1452-1519)*



CODICE ATLANTICO *(1478 ÷ 1518)*



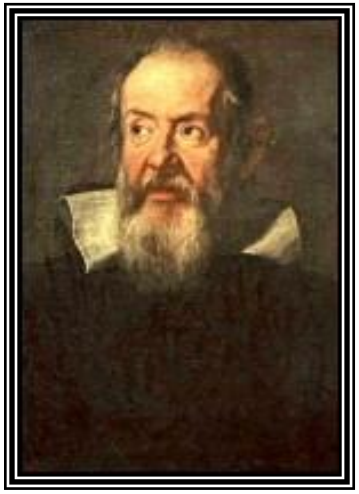


Effetti dimensionali sulla resistenza delle funi …

… prova di trazione

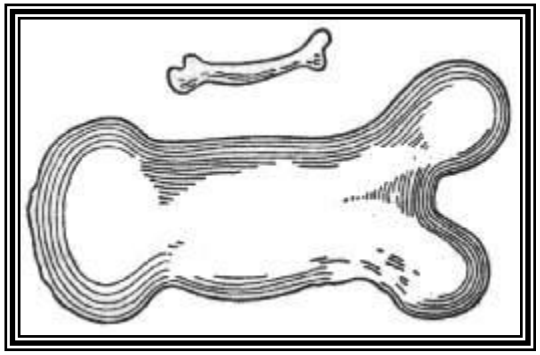
… di poi fa esso filo più corto la metà di prima, e nota quanto peso sostiene di più …

GALILEO GALILEI *(1564-1642)*



DISCORSI E DIMOSTRAZIONI MATEMATICHE INTORNO A DUE NUOVE SCIENZE ATTINENTI ALLA MECANICA ED I MOVIMENTI LOCALI *(LEIDEN, 1638)*


DISCORSI
E
DIMOSTRAZIONI
MATEMATICHE,
intorno à due nuoue scienze
Attenenti alla
MECANICA & i MOVIMENTI LOCALI,
del Signor
GALILEO GALILEI LINCEO,
Filosofo e Matematico primario del Sereniſſimo
Grand Duca di Toſcana.
Con vna Appendice del centro di grauità d'alcuni Solidi.
IN LEIDA,
Appreſſo gli Elſeuirii. M. D. C. XXXVIII.



Effetti dimensionali …

… sulle ossa

Or vegghino come dalle cose sin qui dimostrate apertamente si raccoglie l'impossibilità del poter non solamente l'arte, ma la natura stessa, crescer le sue macchine a vastità immensa …

Problem statement







Cement Water Aggregates



Superplasticizer



Admixtures



Fibers (eventually)



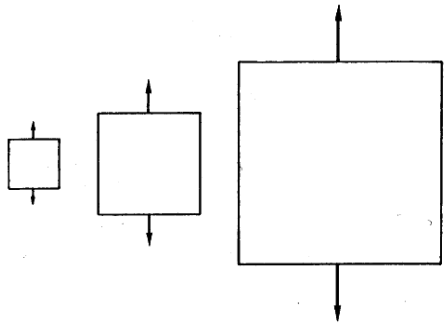
High Performance Concrete (HPC) Normal Strength Concrete (NSC)

La massima dimensione degli aggregati per un calcestruzzo normale era compresa tra 10 mm e 32 mm.
Negli utlimi trenta anni il valore superiore si è ridotto a 20 mm.

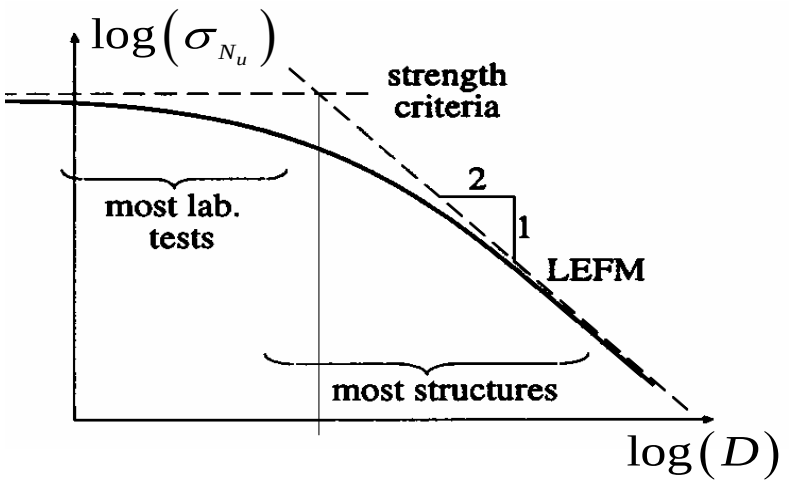
Per le dighe la dimensione massima varia da 40 mm a 200 mm e oltre.

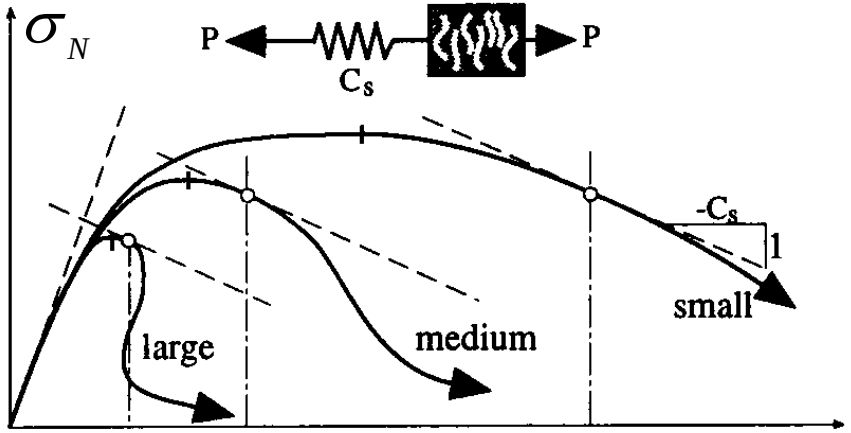
Le dimensioni dei provini per la caratterizzazione meccanica devono avere dimensione minima di 4 volte il diametro massimo dell'aggregato.

Effetti di scala evidenziati dal confronto delle risposte di strutture geometricamente simili ma di dimensioni differenti:



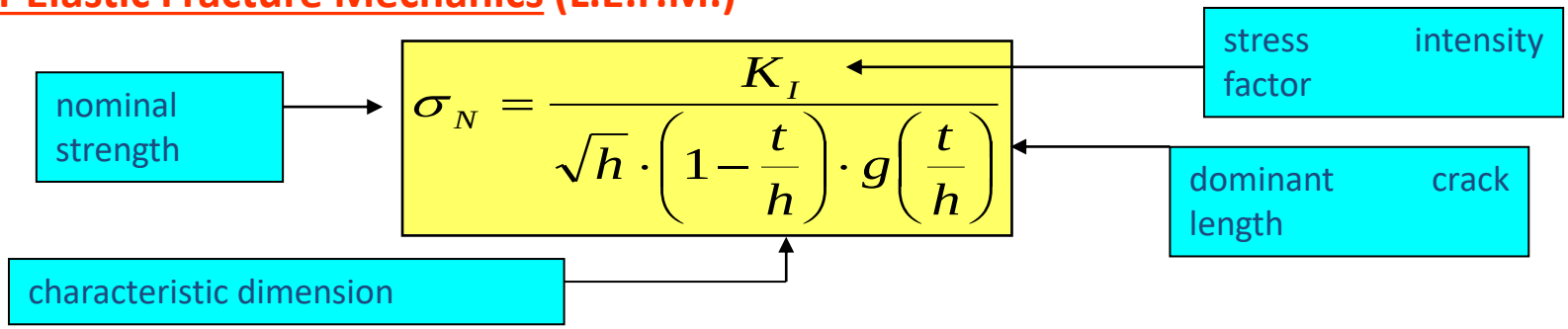
$$\sigma_{N_u} = \frac{P_u}{b \cdot D}$$



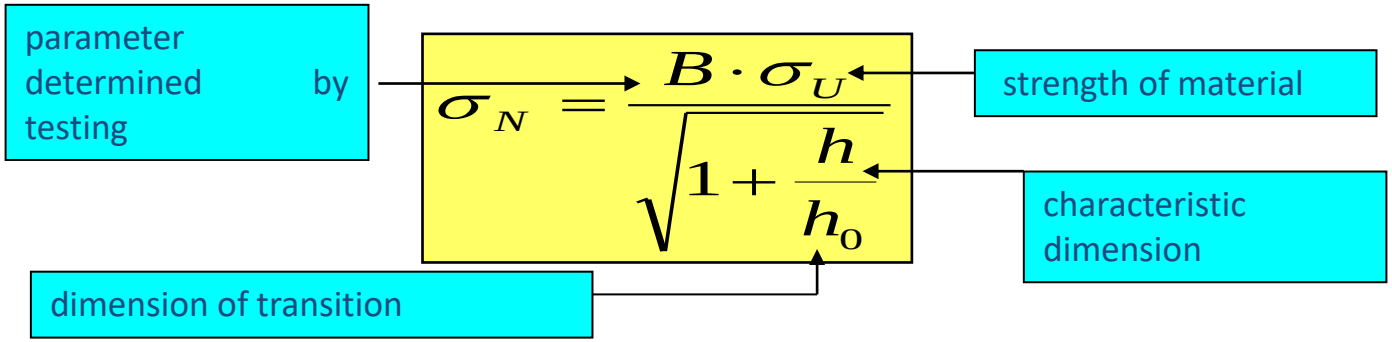


Effetti di scala modelli

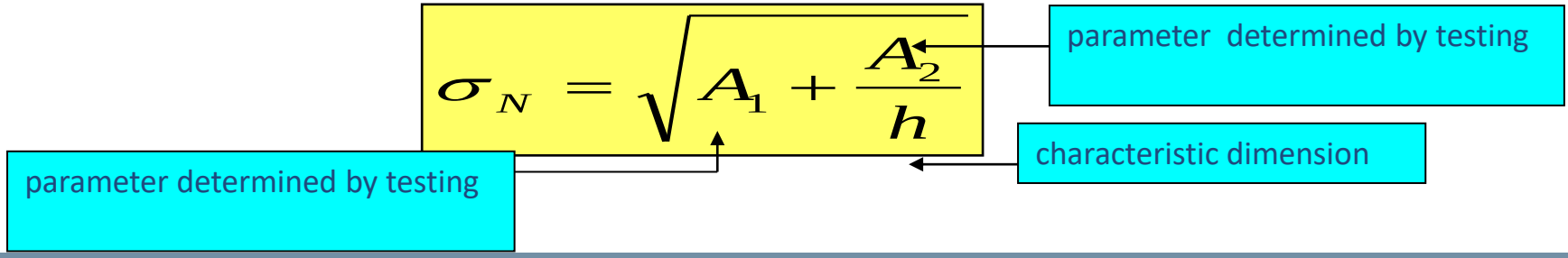
Linear Elastic Fracture Mechanics (L.E.F.M.)



Bažant (1984) ⇒ Size Effect Law (S.E.L.)



Carpinteri (1994) ⇒ Multifractal Scaling Law (M.F.S.L.)



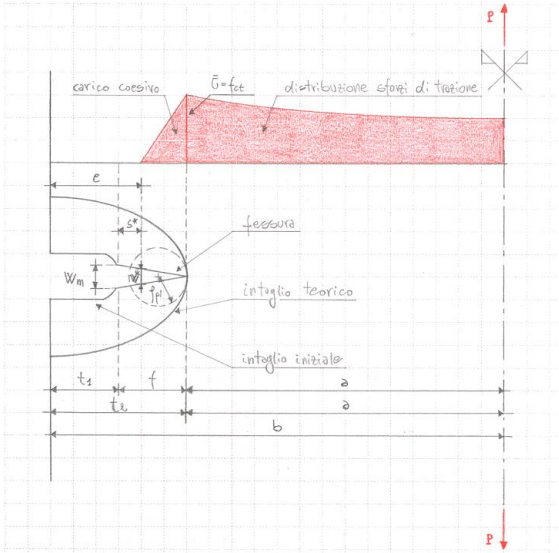
This theory assumes that:

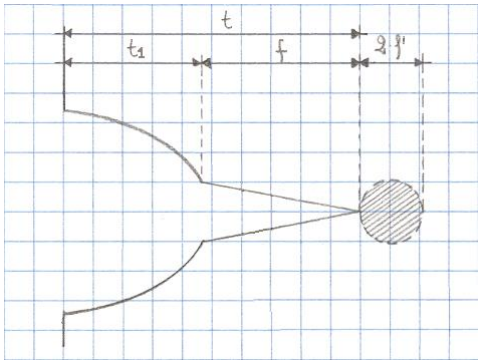
- A dominant crack grows at failure.
- The crack is equivalent to a pointed notch with a small plastic area at the apex (radius ρ').

The Neuber's solution is valid for bending, tension-compression and shear.

The solution is a particular case of general solution (closed form) of stress function (plane stress) for notch contour loaded with cohesive stresses (Rosati and Gelmini 2005)

Il Modello





$$\sigma_N = \frac{\sigma_U}{1 + \frac{c_1}{\sqrt{\rho'}} \cdot \sqrt{\frac{(h-t)\cdot t}{(h-t) + c_2{}^2 \cdot t}}}$$

parameter depending on generalized stress

constitutive strength of material

dominant crack length

characteristic dimension

Parameter depending on generalized stress

Espressioni del difetto dominante

(A)

$$\sigma_N = \frac{\sigma_U}{1 + \frac{c_1}{\sqrt{\rho'}} \cdot \sqrt{h} \cdot \sqrt{\frac{(1-\xi)\cdot\xi}{(1-\xi) + c_2^{\,2} \cdot \xi}}}$$

the ratio ξ = t/h = constant

(B)

$$\sigma_N = \frac{\sigma_U}{1 + \frac{c_1}{\sqrt{\rho'}} \cdot \sqrt{\frac{(h-t)\cdot t}{(h-t) + c_2^{\,2} \cdot t}}}$$

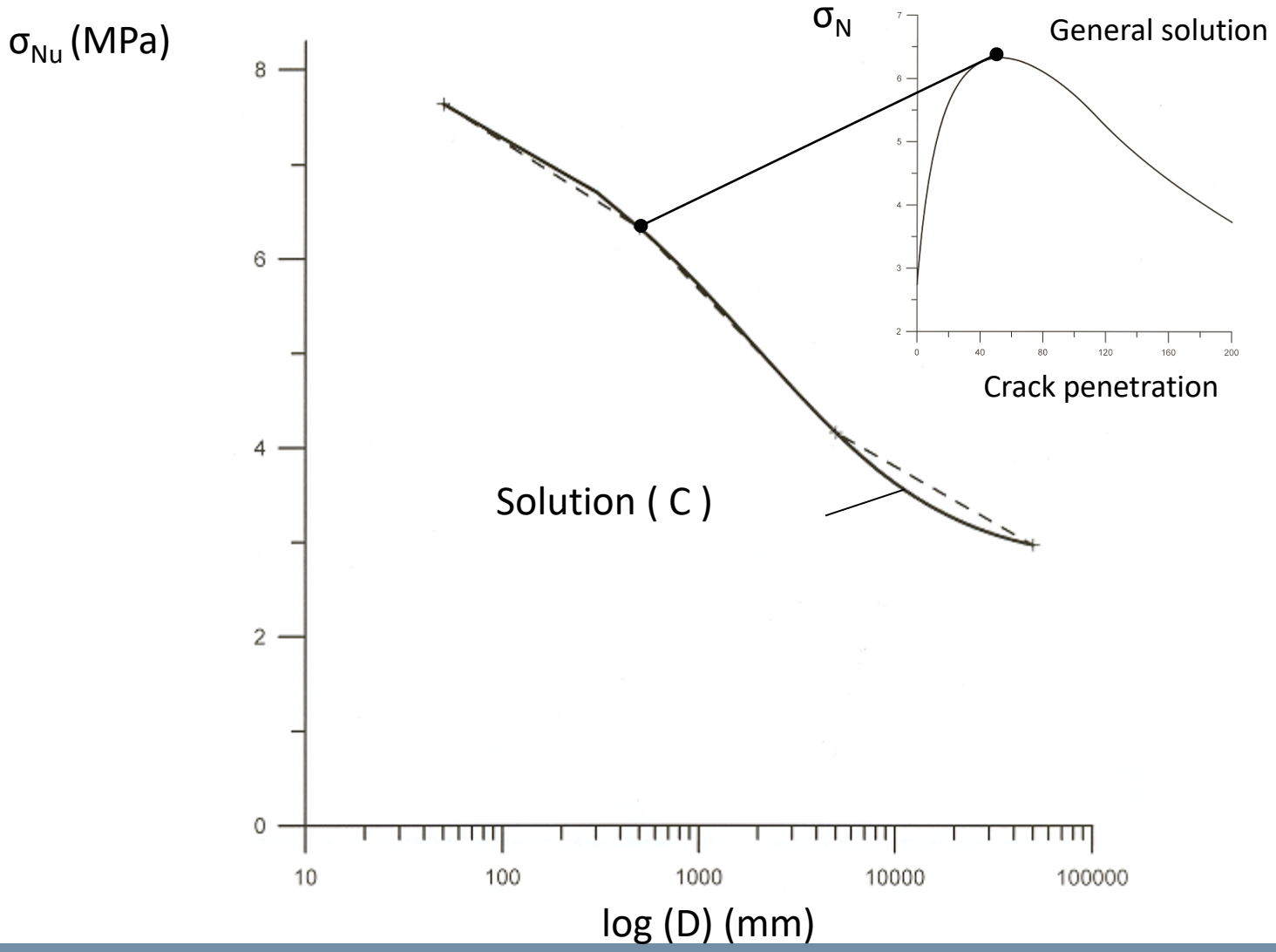
Assuming the crack length t
independent of h

(C)

$$\sigma_N = \frac{\sigma_U}{1 + \frac{c_1\sqrt{t_{\max}}}{\sqrt{\rho'}} \cdot \sqrt{\frac{\left(h - t_{\max}\left(1 - \frac{c}{h+c}\right)\right)\left(1 - \frac{c}{h+c}\right)}{h - \left(1 - c_2^{\,2}\right)t_{\max}\left(1 - \frac{c}{h+c}\right)}}}$$

$t = t_{max}(1\text{-}c\ /(h+c))$
t_{max} is the maximum crack length
c experimental parameter.

Il Modello: la legge di scala

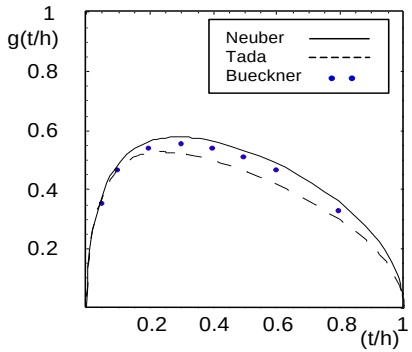


Confronto tra formulazione A e la Meccanica della frattura elastica lineare: sensibilità all'intaglio

Considering the solution of a single edge notch subjected to bending (for example four point bending) the expression provides the nominal strength as a function of h

$$\sigma_N = \frac{\sigma_U}{1+\frac{1}{\sqrt{\rho'}}\sqrt{h}\sqrt{\frac{c_1^{\,2}\left(1-\frac{t}{h}\right)}{\left(\frac{h}{t}-1\right)+c_2^{\,2}}}}$$

(A)



Stress intensity factors for notched beams (Neuber, Tada and Bueckner).

for $h \to \infty$

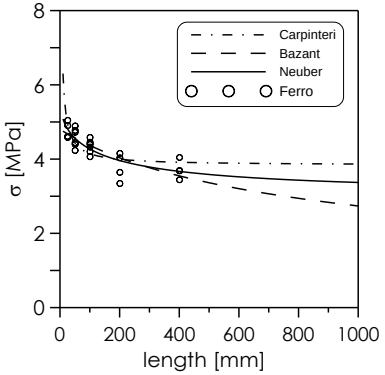
$$\sigma_N = \frac{K_I}{\sqrt{h}\cdot\left(1-\frac{t}{h}\right)\cdot g\left(\frac{t}{h}\right)}$$

$$\sigma_N = \frac{\sigma_U}{\frac{1}{\sqrt{\rho'}}\sqrt{h}\cdot f_N\left(\frac{t}{h}\right)}$$

$$\sqrt{\rho'} = \frac{K_I}{\sigma_U}$$

with $$f_N\left(\frac{t}{h}\right) = \frac{c_1^{\,2}\left(1-\frac{t}{h}\right)}{\left(\frac{h}{t}-1\right)+c_2^{\,2}} \approx \left(1-\frac{t}{h}\right)\cdot g\left(\frac{t}{h}\right)$$

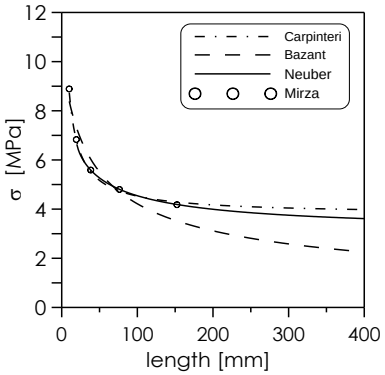
Confronto con I risultati sperimentali.



Coefficient
of determination
R-squared
Carpinteri: 0.585
Bažant: 0.670
Neuber: 0.716

FERRO (1993),direct tension

S.E.L. M.F.S.L.
FORMULATION C

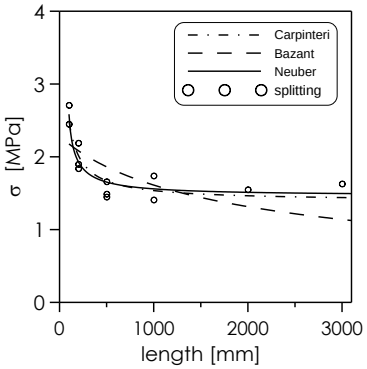


Coefficient of
determination
R-squared
Carpinteri: 0.998
Bažant: 0.919
Neuber: 0.999

SABNIS - MIRZA (1979), 4-point bending

S.E.L. M.F.S.L.
FORMULATION C

Best fitting



Coefficient
of determination
R-squared
Carpinteri: 0.837
Bažant: 0.381
Neuber: 0.866

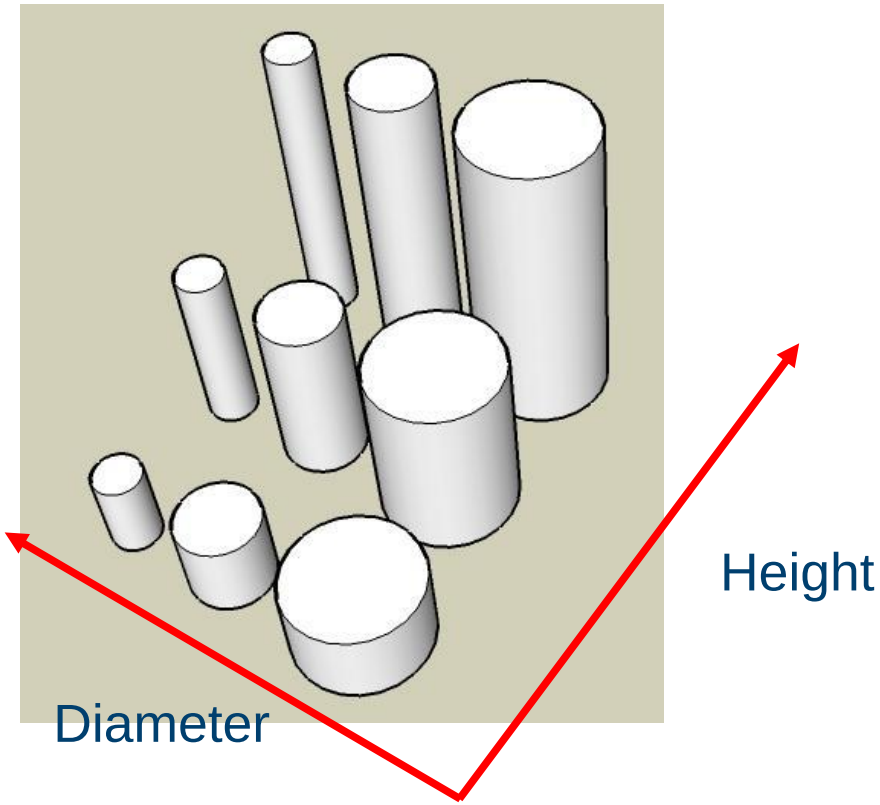
HASEGAWA (1985), splitting

S.E.L. M.F.S.L.
FORMULATION C

Experimental investigation /1

Fracture Process Zone may be a function of two dimensions:

a. height (h)
b. diameter (D)



Experimental investigation /2

- Concrete cylinder of circular cross sections of diameter (D) 100, 200, 400 and 800 mm.
- Height (h) of cylindrical specimens 200, 400, 800, 1600 and 3200 mm.
- The above combinations resulted in three specimen slenderness ratios (λ) of 4, 8, and 32

- The specimens were provided with no steel reinforcements.
- Concrete mix proportions (by weight) were 1/0.14/2.47/1.33/0.41 corresponding to cement/limestone filler/8 mm fine aggregate/20mm crushed coarse/water.
- Cement CEM II/A-LL 42.5R. The maximum coarse aggregate size was 20 mm.



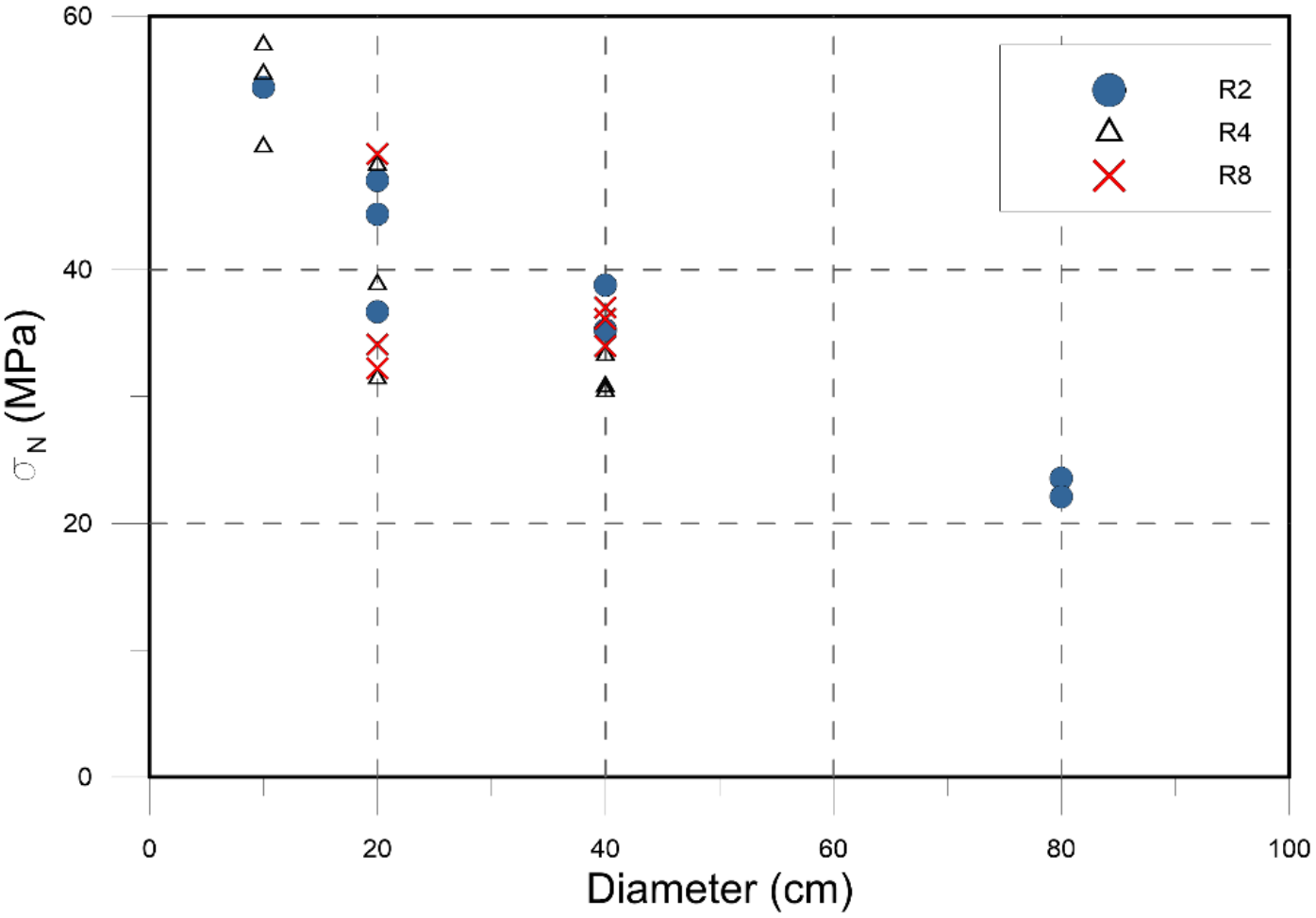
Test setup







Test results /1



Test results /2





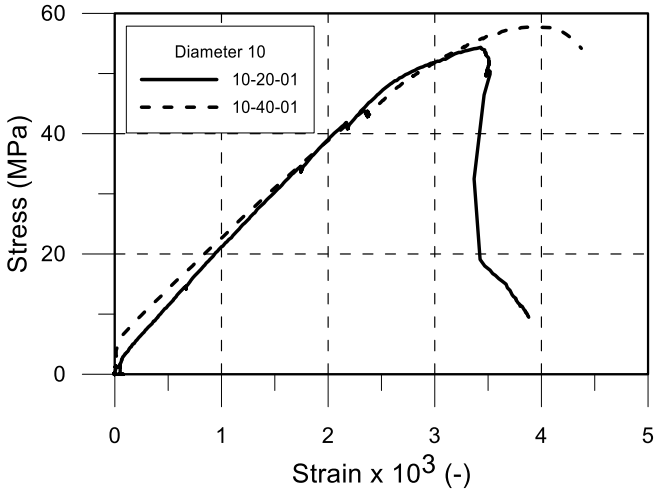
Test results /3

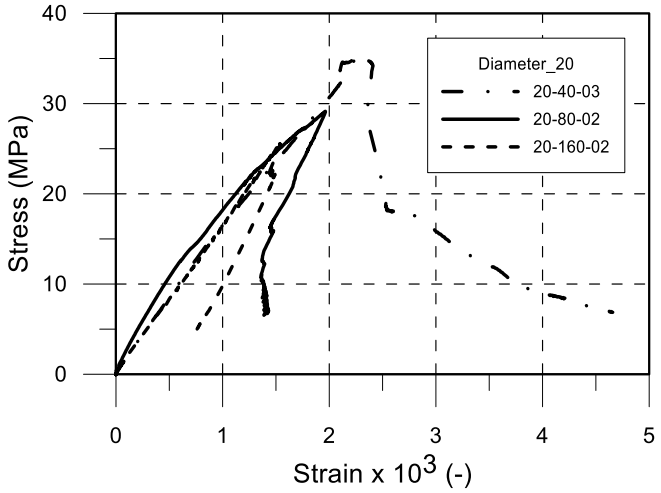


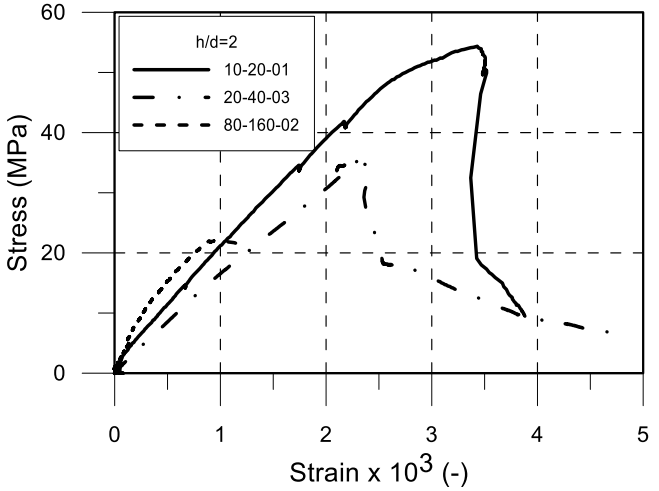


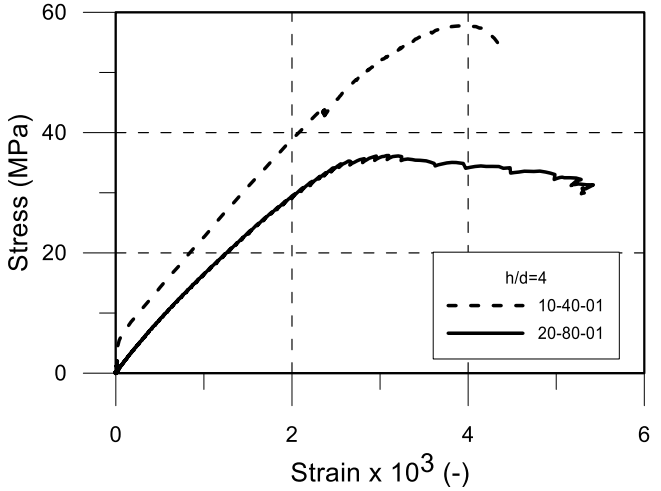


Test results /4





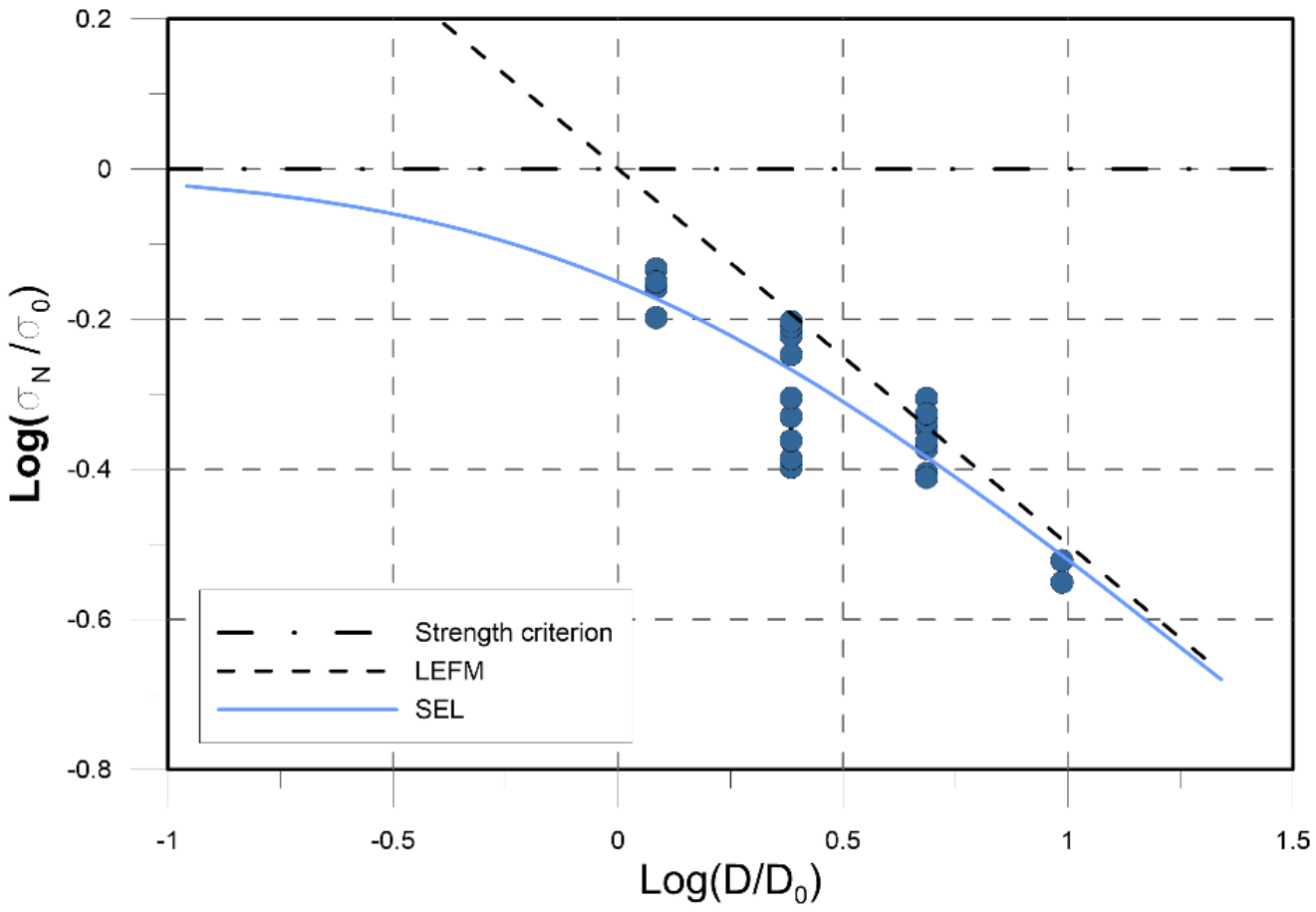




Effect of size

$$\sigma_N = \sigma_0 \cdot \left(1 + \frac{D}{D_0}\right)^{-1/2}$$

s_0 = 78.41 MPa, D_0 = 8.24 cm



Conclusions

1. The variation in the nominal compressive strength of concrete due to the size of the specimen was experimentally investigated in the range of diameter 10cm to 80cm and in the range of length over diameter slenderness ratio 2 to 4 (up to a maximum eight of 320cm).

2. It was found that, for a given value of diameter, the variation in the nominal compressive strength when increasing the slenderness ratio is not substantial. The scatter in the results is more evident for diameter 20cm, this being probably due to the different main crack (band or axial splitting) detected for such size at any slenderness ratio.

3. When increasing the absolute size of the specimen (i.e. the diameter) a clear dependence of the nominal compressive strength on the specimen size is observed. Such effect should consequently be accounted in existing design codes in all the cases where the reinforcement ratio and detailing is not able to control the crack propagation.